**Lecture on the anomalous diffusion in Condensed Matter Physics**

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 Diffusion is a natural or artificial process that governs many phenomena in nature, and enters in various industrial formulations (processed foods agrochemical products, pharmaceutical preparations, personal care goods…). The most known diffusion is the Brownian motion, where the mean-distance-displacement followed by the tracer (diffusive particle among others) increases as the square-root of time. It is not the case, however, for complex systems, where the diffusion is rather slow, because at small-scale, these media present a heterogenous structure. This kind of slow motion is called subdiffusion in literature, and the mean-square-displacement reads

$W\left(t\right)=\left〈\left[\vec{r}\left(t\right)-\vec{r}\left(0\right)\right]^{2}\right〉=2D\_{α}t^{α}$ , $0<α<1$

which deviates from the linear dependence on time found for the Brownian motion. Here, $\vec{r}\left(t\right) $accounts for the position in time of the random walker, and $D\_{α}$ for the generalized diffusion constant, also called "fractional diffusion coefficient". We note that the subdiffusion is a characteristic of the *crowded* system, where the trajectories of their mobile entities are strongly correlated. The above scaling relation is valid for large-times, that is beyond some characteristic time that depends on the specific details of the diffusion process and the structure of the host medium. Generally, a particle is said to be subdiffusive if the condition $W\left(t\right)/t\rightarrow 0$, for $t\rightarrow +\infty $, is fulfilled (very slow diffusion). This explains why the exponent $α$ must be in the interval $0<α<1$.

 The subdiffusive transport is encountered in a variety of systems including the random-walk in fractal structures, fractional-time Brownian motion, living systems, charge carrier transport in amorphous semiconductors, NMR diffusometry on percolation structures, and the motion of a colloid in a polymer network. For example, for diffusion in fractal structures, $α=2/d\_{w}$, where $d\_{w}>2$ is the *random walk-dimension* ($d\_{w}=2d\_{f}/d\_{s}$, where $d\_{f}$ and $d\_{s}$ are the fractal and spectral dimensions, respectively), and for the fractional-time Brownian motion, $α=2H$, where $H$ is the *Hurst index*. Examples of enhanced (or faster) diffusion $\left(α>1\right)$ include tracer particles in *vortex arrays* in a *rotating flow*, *layered velocity fields*, and *Richardson diffusion*. The case $1<α<2$ refers to superdiffusion (turbulent plasmas, Levy-flights, transport in polymers), $α=2$, to *ballistic diffusion* (optical traps), and $α=3$, to Richardson diffusion (atmospheric turbulence). The subdiffusion or superdiffusion exponent $α$ is not a universal quantity, but mainly depends on the nature of elementary constituents.

In this conference, we report on new trends dealt with anomalous diffusion within complex systems within Condensed Matter Physics, from experimental and theoretical points of views.